

CHAPTER THREE

INEQUALITIES IN ONE VARIABLE

Symbols used:

-There are four symbols used and these are

1. $<$ 2. \leq 3. $>$ 4. \geq

The meaning of the symbols $<$

Less than $<$ greater than, greater than $>$ less than.

For these two symbols, the sharp or pointed edge always points to the less or small value, while the other side points towards the great or big value.

Q1. Give the meaning of the following inequalities:

(a) $x < 4$

Solution

X is less than 4 or 4 is greater than x.

(b) $2 < 4$

Solution

2 is less than 4 or 4 is greater than 2.

(c) $y > x$

Solution

y is greater than x or x is less than y.

(d) $5 > 2$

Solution

5 is greater than 2 or 2 is less than 5.

(e) $9x > 10$

Solution

9x is greater than 10 or 10 is less than 9x.

The meaning of the symbol \leq and \geq

1. greater than or equal to \geq less than or equal to

2) less than or equal to \leq greater than or equal to

- With respect to these two symbols, the sharp edge points towards the less than or equal to value, while the other side points towards the greater than or equal to value.

Q2. Given the meaning of the following inequalities:

(a) $b \leq 5$

Solution

b is less than or equal to 5, or 5 is greater than or equal to b.

(b) $x \leq 2$

Solution

x is less than or equal to 2, or 2 is greater than or equal to x.

(c) $5 \leq y$

Solution

5 is less than or equal to y or y is greater than or equal to 5.

(d) $4 \geq y$

Solution

Y is less than or equal to 4 or 4 is greater than or equal to y.

(e) $2x \geq 6$

Solution

6 is less than or equal to 2x or 2x is greater than or equal to 6.

Q3. List the members of the following sets:

(a) $Z = \{x : x > 2\}$

Solution

$x : x > 2 \Rightarrow x$ is greater than 2. The members are all the numbers greater than 2
 $\Rightarrow Z = \{3, 4, 5, 6, \dots\}$

(b) $Y = \{x : x \geq 2\}$

Solution

$x \geq 2 \Rightarrow x$ is greater than or equal to 2,
 $\Rightarrow Y = \{2, 3, 4, 5, \dots\}$

(c) $A = \{n : n > 5\}$

Solution

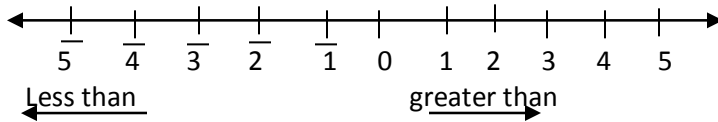
$n > 5 \Rightarrow n$ is greater than 5
 $\Rightarrow A = \{6, 7, 8, 9, \dots\}$

(d) $N = \{n : n \geq 5\}$

Solution

$N \geq 5 \Rightarrow n$ is greater than or equal to 5,
 $\Rightarrow N = \{5, 6, 7, 8, \dots\}$

NB:



-The numbers on the left hand side of the number line, are always less than those on the right hand side.

For examples

1. -5 is less than -4
2. -5 is less than -2
3. -4 is less than -2
4. -4 is less than -3
5. -1 is greater than -3
6. -2 is greater than -5

Q4. List the members of the following sets:

(a) $X = \{x : x \leq 2\}$

Solution

$x \leq 2 \Rightarrow x$ is less than or equal to 2.
 $\Rightarrow X = \{2, 1, 0, -1, -2, -3, \dots\}$.

(b) $Y = \{x : x < 2\}$

Solution

$x < 2 \Rightarrow x$ is less than 2,
 $\Rightarrow Y = \{1, 0, -1, -2, -3, \dots\}$.

(c) $Y = \{n : n \leq -2\}$

Solution

$n \leq -2 \Rightarrow n$ is less than or equal to -2.
 $\Rightarrow Y = \{-2, -3, -4, -5, \dots\}$.

(d) $M = \{n : n < -2\}$

Solution

$n < -2 \Rightarrow n$ is less than -2.
 $\Rightarrow M = \{-3, -4, -5, -6, \dots\}$.

(e) $Z = \{x : x > -4\}$

Solution

$x > -4 \Rightarrow x$ is greater than -4,
 $\Rightarrow Z = \{-3, -2, -1, 0, 1, 2, \dots\}$

(f) $Z = \{x : x \geq -4\}$

Solution

$x \geq -4 \Rightarrow x$ is greater than or equal to -4
 $\Rightarrow Z = \{-4, -3, -2, -1, 0, 1, 2, 3, \dots\}$.

(g) $Y = \{x : x \leq -4\}$

Solution

$x \leq -4 \Rightarrow x$ is less than or equal to -4
 $\Rightarrow Y = \{-4, -5, -6, -7, \dots\}$.

(h) $Y = \{x : x < -4\}$

Solution

$x < -4 \Rightarrow x$ is less than -4
 $\Rightarrow Y = \{-5, -6, -7, \dots\}$.

Q5. Determine the members of each of the following given inequalities:

(a) $Y = \{x : 2 < x < 5\}$

Solution

$2 < x < 5 \Rightarrow x$ is greater than 2 but less than 5
 $\Rightarrow Y = \{3, 4\}$.

(b) $Y = \{x : 2 \leq x \leq 5\}$

Solution

$2 \leq x \leq 5 \Rightarrow x$ is greater than or equal to 2, and x is less than or equal to 5.
 $\Rightarrow Y = \{2, 3, 4, 5\}$.

(c) $X = \{x : 2 \leq x < 5\}$

Solution

$2 \leq x < 5 \Rightarrow x$ is greater than or equal to 2 and x is less than 5.
 $\Rightarrow X = \{2, 3, 4\}$.

(d) $M = \{x : 2 < x \leq 5\}$

Solution

$2 < x \leq 5 \Rightarrow x$ is greater than 2 and x is less than or equal to 5.
 $\Rightarrow M = \{3, 4, 5\}$.

(e) $Y = \{x : -2 \leq x \leq 4\}$

Solution

$-2 \leq x \leq 4 \Rightarrow x$ is greater than or equal to -2 , and x is less than or equal to 4.
 $\Rightarrow Y = \{-2, -1, 0, 1, 2, 3, 4\}$.

(f) $Y = \{x : -2 \leq x < 4\}$

Solution

$-2 \leq x < 4 \Rightarrow x$ is greater than or equal to -2 and x is less than 4.

$$\Rightarrow Y = \{-2, -1, 0, 1, 2, 3\}.$$

$$(g) Y = \{x: -2 < x \leq 4\}$$

Solution

$2 < x \leq 4 \Rightarrow x$ is greater than 2 and less than or equal to 4.

$$\Rightarrow Y = \{-1, 0, 1, 2, 3, 4\}.$$

$$(h) Z = \{n: -6 \leq n \leq -2\}$$

Solution

$-6 \leq n \leq -2 \Rightarrow n$ is greater than or equal to -6 and n is less than or equal to -2.

$$\Rightarrow Z = \{-6, -5, -4, -3, -2\}$$

$$(i) Z = \{n: -6 \leq n < -2\}$$

Solution

$-6 \leq n < -2 \Rightarrow n$ is greater than or equal to -6 and n is less than or equal to -2.

$$\Rightarrow Z = \{-6, -5, -4, -3\}.$$

$$(j) Z = \{n: -6 < n \leq -2\}$$

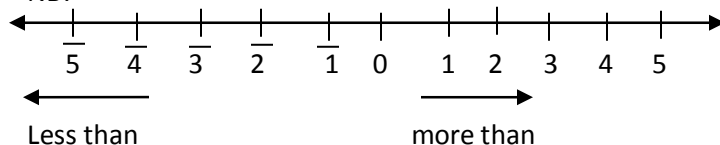
Solution

$-6 < n \leq -2 \Rightarrow n$ is greater than -6 and less than or equal to -2.

$$\Rightarrow Z = \{-5, -4, -3, -2\}.$$

Graphs of inequalities:

NB:



- In the graphical representation of inequalities, if the sign is greater than, then movement is in the right hand side direction.

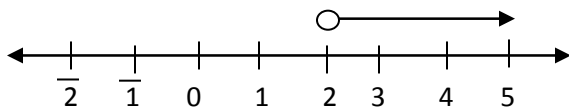
- If we are dealing with the less than sign, then we move in the left hand side direction as indicated above.

Q1. Represent these inequalities graphically or by means of graphs:

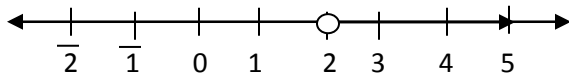
$$a. x > 2$$

Solution

$$x > 2 \Rightarrow x \text{ is greater than } 2. \text{ i.e } \{3, 4, 5, 6, \dots\}$$



or



NB: If the circle is not shaded, then number below it or within it is not a member of the set.

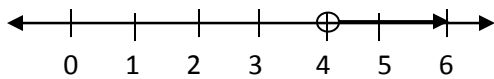
b. $x > 4$

Solution

$x > 4 \Rightarrow x$ is greater than 4 i.e. $\{5, 6, 7, 8, \dots\}$



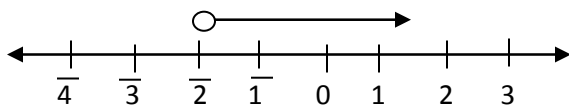
or



c. $Y = \{x : x > -2\}$

Solution

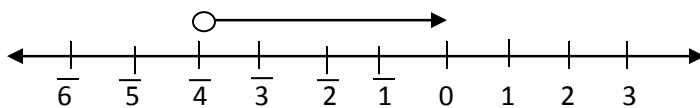
$x > -2 \Rightarrow x$ is greater than -2



d. $X = \{n : n > -4\}$

Solution

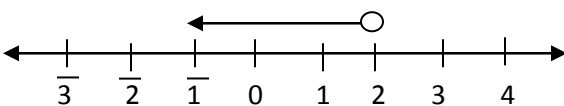
$n > -4 \Rightarrow n$ is greater than -4



e. $Y = \{x : x < 2\}$

Solution

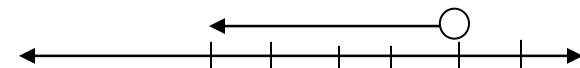
$x < 2 \Rightarrow x$ is less than 2



f. $x = \{n : n < 4\}$

Solution

$n < 4 \Rightarrow n$ is less than 4



0 1 2 3 4 5